

CERN-TH/96-278  
hep-ph/9610234

# TOPICS IN JET PHYSICS<sup>1</sup>

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## Abstract

We review two subjects in the theoretical study of jet production at the Tevatron collider: the uncertainties in the determination of the partonic densities inside the proton and the uncertainties in the calculation of higher-order corrections to the QCD matrix elements.

CERN-TH/96-278  
October 1996

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<sup>1</sup>To appear in the Proceedings of the XIth Topical Workshop on Hadron Collider Physics, Abano Terme, Padova, Italy, May 1996.

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# 1 Introduction

The accurate measurements of inclusive jet distributions recently reported by the experiments at the Fermilab Tevatron collider [1, 2] pose a serious challenge to the theorists' capability to evaluate the relative cross sections with matching precision. The current data, and in particular the discrepancy between data and theory at high  $E_T$  reported by CDF [1], call for an improved assessment of the theoretical uncertainties present in the evaluation of the 1-jet inclusive  $E_T$  distributions. Several groups have undertaken this task, contributing in the past several months to a significant progress in the field. The duty assigned to me by the organizers of the Workshop was to review this progress.

We can divide the problem into two main subjects: (i) the study of the uncertainties in the determination of the partonic densities inside the proton (discussed here in Section 2), and (ii) the study of the uncertainties in the perturbative calculation of the QCD matrix elements (discussed in Section 3). I will skip all preliminaries on jet definitions and technical aspects of the evaluation of the NLO matrix elements, assuming that the reader is familiar with them. For a complete discussion of these points the reader is referred to the references quoted below.

## 2 Parton Densities

The cross section for 1-jet inclusive production is known today up to NLO accuracy [3, 4, 5]. It is very unlikely that full NNLO results will become available in the short term, due to the complexity of the calculations. A standard way to estimate the possible size of higher-order corrections is to evaluate the dependence of the NLO result w.r.t. variations of the factorization and renormalization scales. The analysis in the range  $50 < E_T(\text{GeV}) < 400$  shows a scale dependence of the order of 10–20% [4, 6]. This uncertainty is similar to the current experimental uncertainties, and its small size makes the comparison between theory and data a compelling test of perturbative QCD, as well as an important probe of the presence of possible new physics.

However accurate the perturbative evaluation of the partonic matrix elements, the ultimate precision of the theoretical calculations for the jet cross sections is limited by the uncertainty in the knowledge of the parton-density functions (PDFs) of the proton. PDF parametrizations are extracted from global fits to a large variety of data for which NLO QCD predictions are available [7]. The most significant inputs are given by deep inelastic scattering (DIS), Drell–Yan (DY) and prompt-photon fixed-target production data. DIS provides a direct probe of the quark densities via the measurement of  $F_2(x, Q^2)$ . It also gives a value of  $\alpha_s$  via the scaling violations of  $F_2$  at intermediate values of  $x$  ( $x \sim 0.3$ – $0.5$ ). In this  $x$  range  $d \log F_2 / d \log Q^2$  receives no significant contribution from the gluon density and  $Q^2$  is still sufficiently large for higher-twist effects to be under control. The contributions of the different quark flavours can be separated by using a mixture of DIS and DY data for different beams and targets. The gluon density at small  $x$  can be extracted from the scaling violations of  $F_2(x, Q^2)$  (for  $x$  values sufficiently small  $dF_2/d \log Q^2 \propto \alpha_s(Q^2)G(x, Q^2)$ ).  $G(x, Q^2)$  is rather well known today in the region  $10^{-4} < x < 0.1$ . The most direct probe on the gluon density at larger values of  $x$  so far has been the measurement of large- $E_T$  prompt photons produced in fixed-target experiments with proton beams, where the Compton channel  $qg \rightarrow q\gamma$  dominates at large  $x$  over the quark-

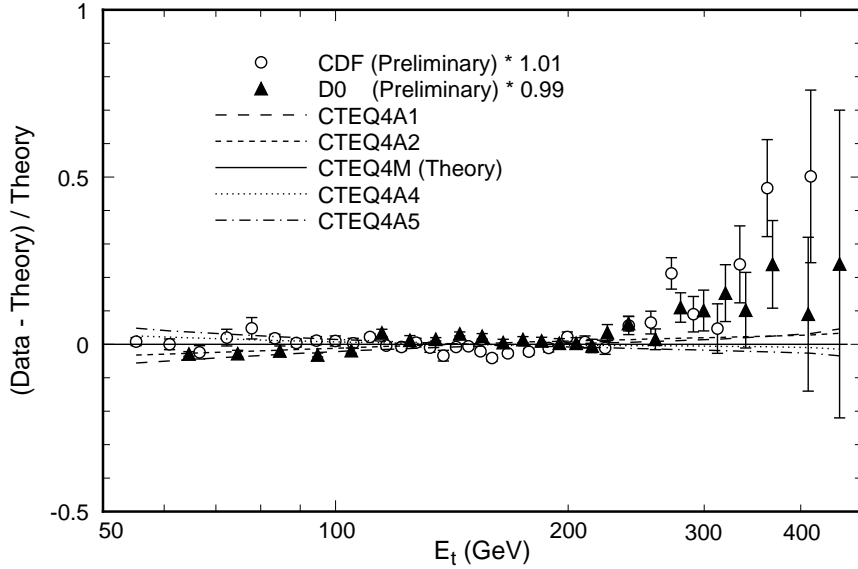


Figure 1: Comparison between NLO QCD predictions for the 1-jet inclusive  $E_T$  distribution and Tevatron data (figure taken from ref. [8]). The PDF sets used belong to the CTEQ4 family [8].

sea-suppressed annihilation channel  $q\bar{q} \rightarrow g\gamma$ .

The use of the NLO QCD matrix elements, together with one of the most recent global PDF fits (CTEQ4 [8]), leads to the comparison of the theoretical prediction with the Tevatron data shown in fig. 1. The surplus rate found in the CDF data [1] (not confirmed by, but nevertheless consistent within uncertainties with the D0 measurement [2]) significantly exceeds the expected theoretical uncertainty, estimated with the study of the scale dependence. Similar conclusions are obtained using independent PDF fits, such as those of the MRS group [9, 10].

Several studies have been carried out to establish to which extent the available data used for the extraction of the PDFs set compelling constraints on the highest- $E_T$  tail of the jet distribution. Before illustrating the results of these studies, I will present a few remarks.

The values of  $x$  that are probed and the flavour content of the initial state in jet production depend on the jet  $E_T$ . In fig. 2a I show the fractional contribution to  $d\sigma/dE_T$  given by the three possible initial-state channels ( $gg$ ,  $qg + \bar{q}g$  and  $qq + q\bar{q}$ ). The  $q\bar{q}$  channel has been separated into the contributions of processes which only admit  $s$ -channel exchange (namely  $q\bar{q} \rightarrow gg$  and  $q_i\bar{q}_i \rightarrow q_j\bar{q}_j$ ), labelled as “QQ annih”, and those which also admit a  $t$ -channel exchange (e.g.  $q_i\bar{q}_i \rightarrow q_i\bar{q}_i$  and  $q_i\bar{q}_j \rightarrow q_i\bar{q}_j$ ), labelled as “QQ  $t$ -channel”. These last processes are seen from the figure to dominate the production at the largest  $E_T$  values observed at the Tevatron, namely  $E_T = 450$  GeV.

The ranges of  $x$  that are relevant to the production of jets at three different energies are shown in fig. 2b. While the distributions as a function of  $x$  peak at the obvious value  $x = x_T \equiv 2E_T/\sqrt{S}$ , long tails are present even at  $x$  values significantly larger than  $x_T$ . As a result, in order to have accurate predictions for jet production at the highest transverse energies observed at the Tevatron,  $E_T \sim 450$  GeV, the PDFs should be known for values of  $x$  up to about 0.6.

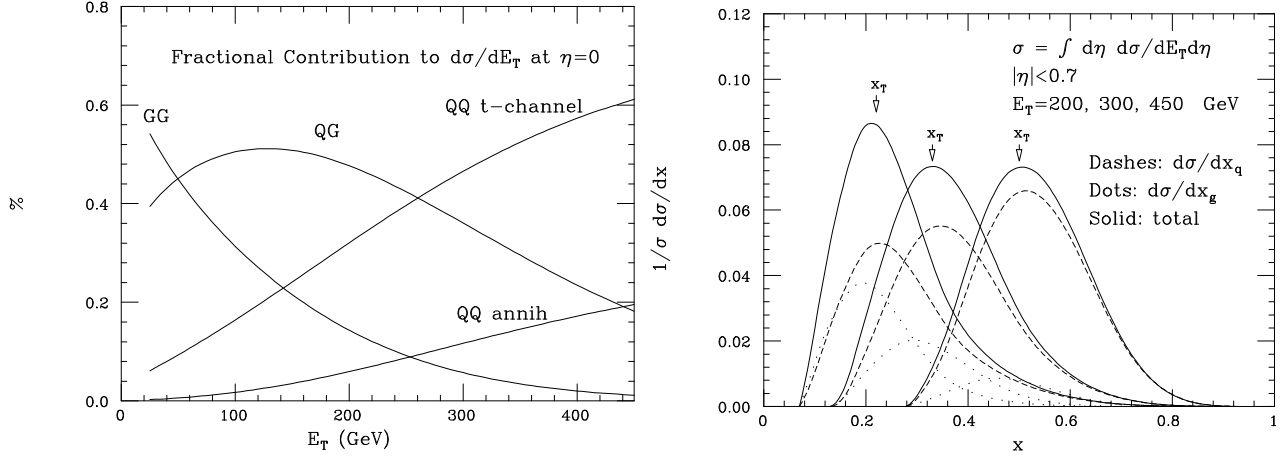


Figure 2: *Left: relative contribution of the different initial states for the production of a jet at  $\eta = 0$  as a function of the jet  $E_T$ . Right: distribution in  $x$  for the production of jets with  $E_T=200, 300$  and  $450$  GeV.*

Since the dominant contribution to the jet rate at 450 GeV is given by quark initial states, it is natural to explore the possibility that the quark densities are not known with sufficient accuracy. First of all, given that the quark densities at large  $x$  are extracted from DIS data at low  $Q^2$ , it is important to establish what is the range in  $x$  at low  $Q^2$  of relevance for the jet rates at high  $Q^2$ . In fact, given the QCD perturbative evolution, it is natural to expect that most of the quarks found at large  $Q^2$  with  $x$  of the order of 0.5 will actually come from quarks having a much larger value of  $x$  at the lowest  $Q^2$  scales explored by the DIS experiments. Figure 3 gives the integral of the quark density above a given value of  $x$  at  $Q = 3$  GeV (a typical scale for DIS data) and at  $Q = 225$  GeV (the scale for production of a 450 GeV jet, when using the standard choice  $\mu_R = \mu_F = E_T/2$ ).

For  $x$  values in the range 0.5–0.6, the evolution reduces by a factor of 3 the probability to find a quark. This in itself is a dramatic change, which prompts some observations. To start with, it is important to point out that the size of the depletion at large  $x$  is controlled by the value of  $\alpha_s$ , since at large  $x$   $d \log F_2 / d \log Q^2 \propto -\alpha_s(Q^2)$ . The larger the value of  $\alpha_s$ , the stronger the depletion. Therefore, although one might expect that a larger value of  $\alpha_s$  would generate an increase in the rate of jet production, the opposite is actually true at large  $E_T$ . This is shown, for example, in fig. 4a, which is taken from the work of ref. [11]. The fit labelled  $J$  (dashed line) is obtained by including the CDF jet data in the global PDF fit. In order to accommodate the shape of the lower-energy jet data, the fit selects a value of  $\alpha_s$  slightly larger than that preferred by the pure DIS analysis ( $\alpha_s(M_Z^2) = 0.120$  vs.  $\alpha_s(M_Z^2) = 0.113$ )<sup>3</sup>. The larger value of  $\alpha_s$  is reflected in the lower rate predicted by this set of PDFs at high  $E_T$ . As an additional comment, it should be remarked that such a significant change in the quark density, induced by the evolution, might be sensitive to the presence of large higher-order corrections to the evolution equations. In the usual analyses, the evolution is performed at NLO, consistently

<sup>3</sup>It is important to notice that this result is consistent with an independent evaluation of  $\alpha_s$  from the CDF jet data [12].

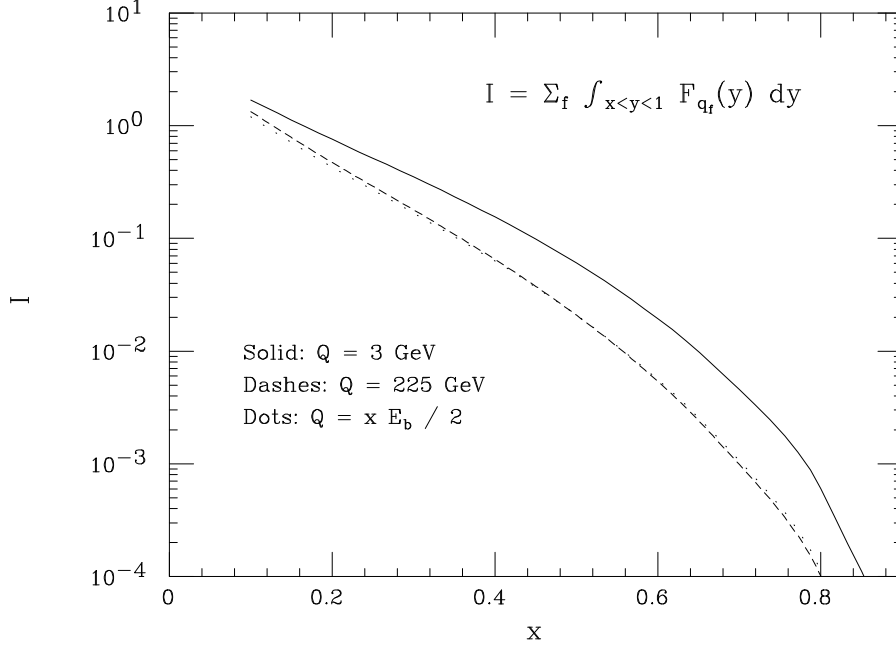


Figure 3: *Evolution of the integrated quark density, summed over all  $q$  and  $\bar{q}$  flavours.*

with the level of accuracy of the matrix element evaluation. In the region of large  $x$  and low  $Q^2$ , however, big corrections could come from the resummation of threshold effects, as suggested in [13]. The inclusion of these effects has never been carried out, and it is hard to evaluate at this stage what their impact could be.

Another interesting aspect of the evolution displayed in fig. 3 is that the number of quarks with  $x > 0.6$  at  $Q = 225$  GeV is approximately equal to the number of quarks with  $x > 0.7$  at  $Q = 3$  GeV. Since the quark density drops very quickly (almost one order of magnitude in going from  $x = 0.7$  to  $x = 0.8$ ), we can conclude that quarks with  $x > 0.7$  at  $Q = 3$  GeV have no significant impact on the density of quarks in the region  $x < 0.6$  at  $Q \sim 200$  GeV. Therefore provided the low-energy DIS data are solid in the range  $x < 0.7$ , we expect no significant uncertainty due to the knowledge of the quark PDFs. The data are indeed quite accurate in this region, as shown in fig. 4b. For this reason, when the jet data are included in a global fit of PDFs the high- $E_T$  points have no impact on the quark densities, which are almost entirely controlled by the small error of the DIS data points.

Figure 4a displays also the results of a fit obtained by removing all DIS data in the region  $x > 0.2$  while including the CDF jet data (fit  $J'$ ). The absence of independent constraints on the quark densities allows the jet data to drive them arbitrarily high, and an acceptable fit of the jet data is possible. However, when this fit is compared with the existing DIS data, the disagreement is absolutely unacceptable. The prediction for  $F_2$  obtained by the  $J'$  fit is shown in fig. 4b, compared with the data and with the results of standard fits. At  $x = 0.55$ ,  $F_2$  is increased by about 20–30%, which is what one would in fact expect in order to accommodate the 50% excess rate observed by CDF. The authors of ref. [11] therefore conclude that it is not possible to fudge the quark densities in order to fit the CDF jet data without an unacceptable

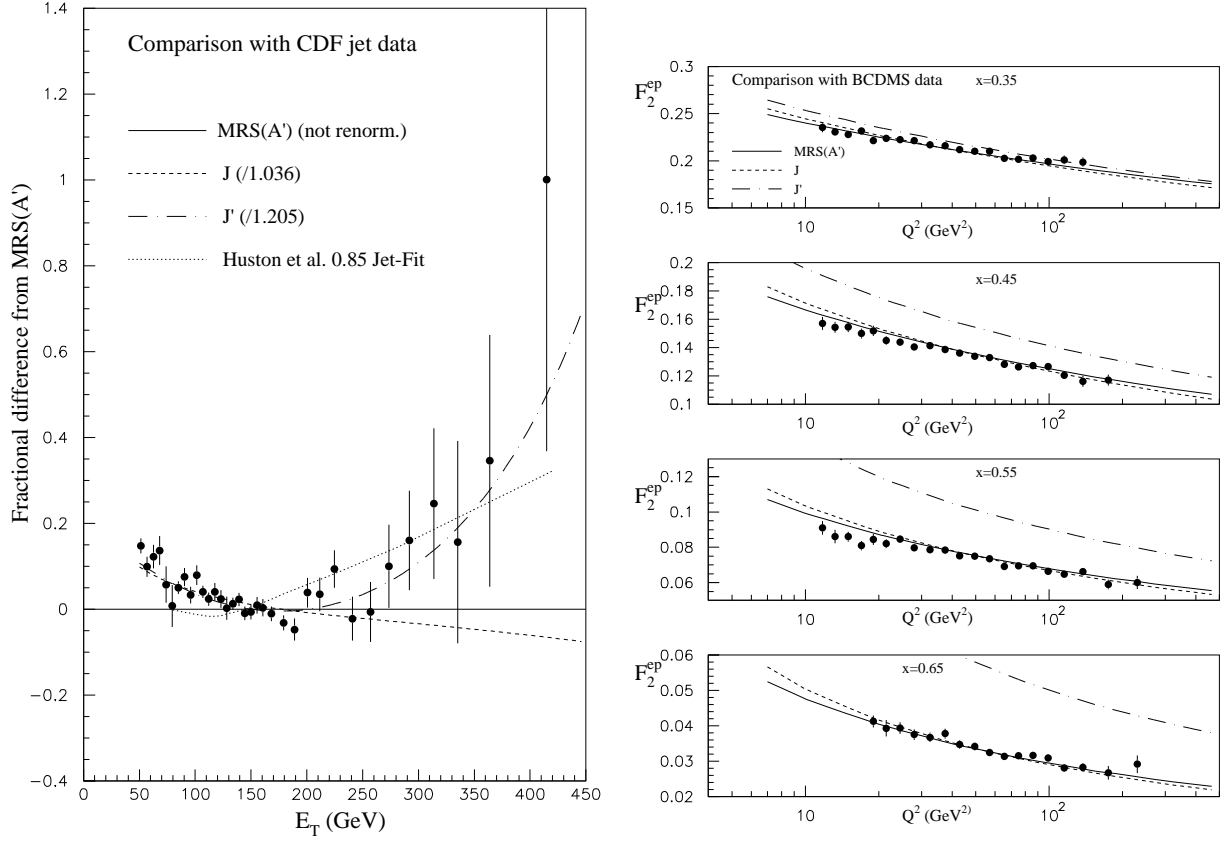


Figure 4: Left: Comparison between NLO QCD predictions for the 1-jet inclusive  $E_T$  distribution and CDF data (figure taken from ref. [11]). The dashed line uses a PDF fit obtained including the CDF jet data in the fit. The dash-dotted line uses a PDF fit which includes the CDF data but leaves out all DIS data for  $x > 0.2$ . Right: Comparison between the PDF fits used in the left figure and BCDMS data for  $F_2$  (figure taken from ref. [11]).

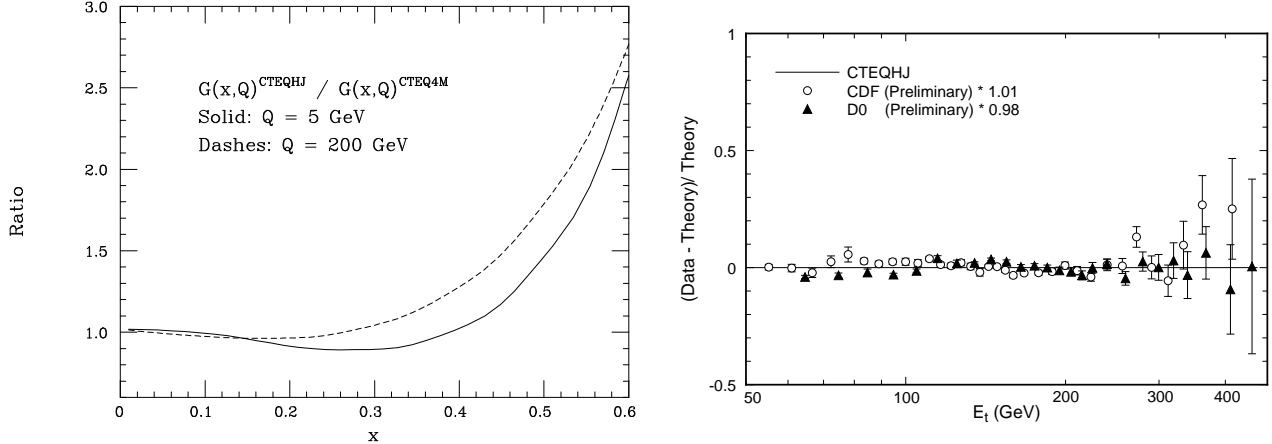


Figure 5: *Left: ratio of the CTEQHJ and CTEQ4M gluon densities. Right: comparison between NLO QCD predictions for the 1-jet inclusive  $E_T$  distribution using the CTEQHJ set and the Tevatron data (figure taken from ref. [8]).*

clash with the high-precision DIS data.

An alternative approach was pursued by the CTEQ group. These authors exploited the fact that the experimental and theoretical uncertainties present in the analysis of the prompt photon data might leave enough room to accommodate larger gluon densities at large  $x$ . From fig. 2 we see that in order for processes with initial-state gluons to explain the 50% excess rate, the gluon density should increase by at least a factor of 2. The CTEQ group performed a global fit [14, 8] including the high- $E_T$  CDF data and assigning to them a larger weight in the  $\chi^2$  determination. The gluon density obtained in this way, called CTEQHJ, is compared with a “standard” gluon density in fig. 5a. The global  $\chi^2$  of the fit is marginally worse than the best  $\chi^2$  (a detailed discussion of this point can be found in ref. [8]). In particular, the theoretical predictions for the prompt-photon spectrum obtained using this set of PDFs are perfectly consistent, within uncertainties, with the experimental data [14]. The comparison between QCD and data for the jet- $E_T$  distributions is shown in fig. 5b. As can be seen, most of the excess rate has been removed by this choice of PDFs.

Before concluding this section I would like to make a few more comments. During the summer, new results on neutrino DIS from the CCFR collaboration were released [15]. The most significant result of these analyses is a new extraction of  $\alpha_s$  from the large- $x$  scaling violations in  $F_2$  and  $xF_3$ . Contrary to all previous extractions of  $\alpha_s$  from DIS [16], this measurement is in perfect agreement with the values of  $\alpha_s$  extracted at higher energies (e.g. at LEP) or in  $\tau$  decays [17]. In particular, the new CCFR measurement of  $\alpha_s(M_Z) = 0.119 \pm 0.005$  is significantly larger than the previous average of DIS measurements,  $\alpha_s(M_Z) = 0.112 \pm 0.004$  [18]. It is hard to evaluate what the impact of these data will be on the calculation of jet cross sections for the Tevatron. The  $F_2$  and  $xF_3$  data have not been released as yet, so that no global fit of PDFs and their evolution is available. It is extremely encouraging that the new DIS  $\alpha_s$  value is consistent with that extracted from the analysis of the jet data [12]. Even more so because these determinations are now consistent with the extraction of  $\alpha_s$  from other independent measurements [19]. It is likely that, because of the enhanced evolution of the large- $x$  valence

densities, the high- $E_T$  CDF anomaly will be even more significant, as we already discussed above. As the CTEQ studies have shown, however, this could be absorbed into an acceptable change in the gluon densities. Independent measurements of the large- $x$  gluon component of the proton are at this point of extreme importance. Higher statistics prompt-photon production data are becoming available [20], and will hopefully help to settle this issue soon.

### 3 Higher orders in perturbation theory

As mentioned in the previous section, the study of the scale dependence in the range  $50 < E_T(\text{GeV}) < 400$  shows an effect of the order of 10–20% [4, 6]. It is reasonable to expect that the NNLO corrections, in this  $E_T$  range, will be of this size. For larger  $E_T$  values the scale dependence becomes more and more pronounced [4, 6], indicating that NNLO effects can no longer be neglected. Whether the actual threshold for the onset of significant NNLO contributions is 400, 450 or 600 GeV, is clearly a question of extreme relevance for the interpretation of the CDF data.

To answer this question we must find a more solid way of estimating the higher-order corrections than just looking at the scale dependence of the lower-order results. Direct estimates of higher-order corrections are possible when these are dominated by terms whose structure does not depend on the specific details of the hard process under consideration. A typical example is given by the large logarithms that are associated to the emission of collinear gluons from the initial state of a hard process. These logarithms are independent of the details of the hard process, and can be resummed to all orders of perturbation theory, giving rise to the standard DGLAP equation which governs the evolution of the parton densities inside the proton.

The resummation of higher-order contributions is generally possible when two different hard scales (say  $\mu$  and  $Q$ , with  $Q \gg \mu$ ) are present in the problem. Gluons whose wavelength is of the order of  $1/\mu$  cannot be sensitive to phenomena taking place at distances of the order of  $1/Q$ . If their emission gives rise to large corrections of order  $\alpha_s \log^k(Q/\mu)$  ( $k \leq 2$ ), as is often the case, these corrections are therefore universal and can be resummed (for a thorough discussion of these issues at NLO, and for a complete set of references to the relevant literature, see refs. [24, 25]). In the case of high-energy jet production, the two different hard scales correspond to the energy of the jet itself and the maximum amount of energy that can be released by initial-state radiation. The closer one gets to the threshold  $M_{jj}^2 \rightarrow S_{had}$  (where  $M_{jj}$  is the invariant mass of the dijet system and  $\sqrt{S_{had}}$  is the total energy in the  $p\bar{p}$  CoM frame), the smaller the amount of energy available for radiation. Given that parton densities fall steeply, the phase space available for soft radiation is already significantly reduced before  $\tau = M_{jj}^2/S_{had}$  approaches 1. In the case of DY production in fixed-target experiments, for example, large corrections are already found for values of  $\tau$  of the order of 0.5 [21, 22]. It is believed that higher powers of  $\alpha_s \log^2(1 - \tau)$  are responsible for the increased sensitivity of the jet cross section at high  $E_T$ , and that these are the leading corrections that need to be resummed to provide a better estimate of the jet cross section beyond NLO.

A technical discussion of how the resummation of threshold corrections is performed, in addition to references to the original literature, can be found in ref. [13]. Here I will limit myself to presenting one result of interest for the study of high-energy jet production. Rather than considering the resummation corrections to the  $E_T$  distribution of jets, I will consider the



case of high-mass dijet pairs. The reason is that resummation is simpler in this case, since for this variable all Sudakov effects are present only in the evolution of the initial state, so that the formalism is almost identical to that of the DY processes. The study of the resummation for the 1-jet inclusive  $E_T$  distribution is in progress.

In fig. 6 I show the following quantities

$$\frac{\delta_{gg}^{(3)}}{\sigma^{(2)}}, \quad \frac{\delta_{qg}^{(3)}}{\sigma^{(2)}}, \quad \frac{\delta_{q\bar{q}}^{(3)}}{\sigma^{(2)}}, \quad \frac{\delta_{gg}^{(3)} + \delta_{qg}^{(3)} + \delta_{q\bar{q}}^{(3)}}{\sigma^{(2)}}, \quad (1)$$

where  $\delta^{(3)}$  is equal to the resummed hadronic cross section in which the terms of order  $\alpha_s^2$  have been subtracted, and  $\sigma^{(2)} = \sigma_{gg}^{(2)} + \sigma_{qg}^{(2)} + \sigma_{q\bar{q}}^{(2)}$  is the full hadronic LO cross section (of order  $\alpha_s^2$ ). I use as a reference renormalization and factorization scale  $\mu = M_{jj}/2$ . Notice that for large invariant masses the effects of higher orders are large. To understand how much is due to the first-order corrections (which are exactly calculable [23, 5]) and how much to corrections of order  $\alpha_s^4$  and higher, I show the following quantities in fig. 7:

$$\frac{\delta_{gg}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{qg}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{q\bar{q}}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{gg}^{(4)} + \delta_{qg}^{(4)} + \delta_{q\bar{q}}^{(4)}}{\sigma^{(3)}}, \quad (2)$$

where  $\delta^{(4)}$  is now equal to the resummed hadronic cross section with terms of order  $\alpha_s^3$  subtracted, and  $\sigma^{(3)}$  is an approximation to the full NL cross section, summed over all subprocesses, obtained by truncating the resummation formula at order  $\alpha_s^3$ . This figure shows that most of the large  $K$  factor is due to the pure NLO corrections, with the resummation of higher-order soft-gluon effects contributing only an additional 10% at dijet masses of the order of 1 TeV. Only above  $M_{jj} > 1200$  GeV, corresponding approximately to  $E_T > 600$  GeV, are the resummation effects non-negligible.

These results should only be taken as an indication of the order of magnitude of the correction, since we have not included here a study of the resummation effects on the determination of the parton densities. As was already mentioned previously, it is quite possible that resummation effects significantly influence the determination of the large- $x$  structure functions from low energy data. Equally important effects could appear in the evolution from low to high  $Q^2$ . It would therefore be important to reexamine the extraction of the large- $x$  non-singlet structure functions, in the light of the resummation results for the DIS process, before firmer conclusions can be drawn on the significance of the present jet cross section discrepancy.

From this preliminary study it seems, however, unlikely that the full 30–50% excess reported by CDF for jet  $E_T$ 's in the range 300–450 GeV could be explained by Sudakov resummation effects in the hard process.

Before concluding this section, I want to mention a puzzling observation recently made in ref. [26]. The authors studied the scheme dependence of the jet cross section, by using the  $\overline{\text{MS}}$  and DIS versions of the same PDF fit, folded with the NLO jet cross section evaluated in the two respective schemes. The two evaluations should yield equal results, up to terms of NNLO. However, the differences turn out to be numerically large, of the order of 40% at the highest  $E_T$ 's. The DIS calculation, in particular, is in good agreement with the CDF data. Whether this difference corresponds to a genuine scheme dependence, and therefore a true uncertainty due to the ignorance on corrections beyond NLO, or whether it is an artefact of the way the DIS PDFs were extracted or evolved, this is still not clear.

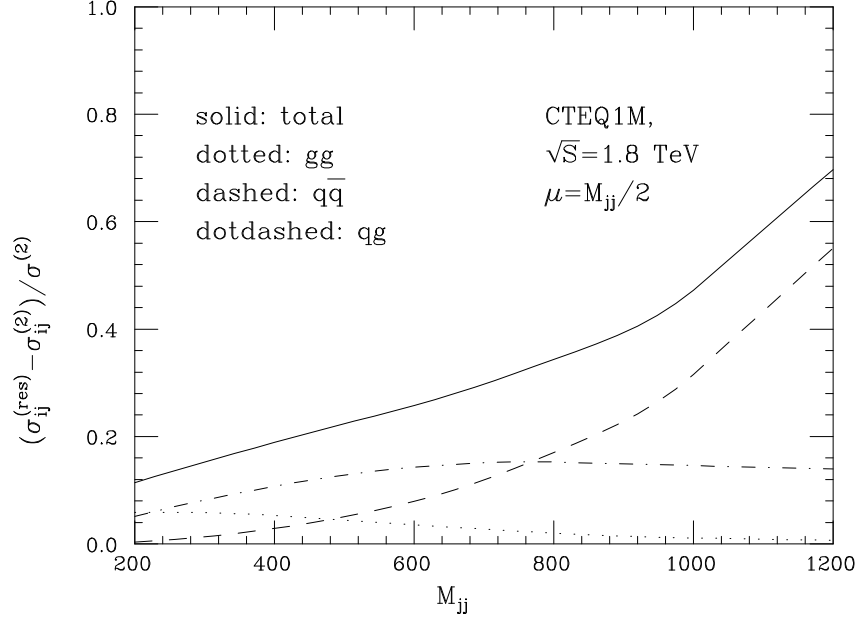


Figure 6: Contribution of gluon resummation at order  $\alpha_s^3$  and higher, relative to the LO result, for the invariant mass distribution of jet pairs at the Tevatron.

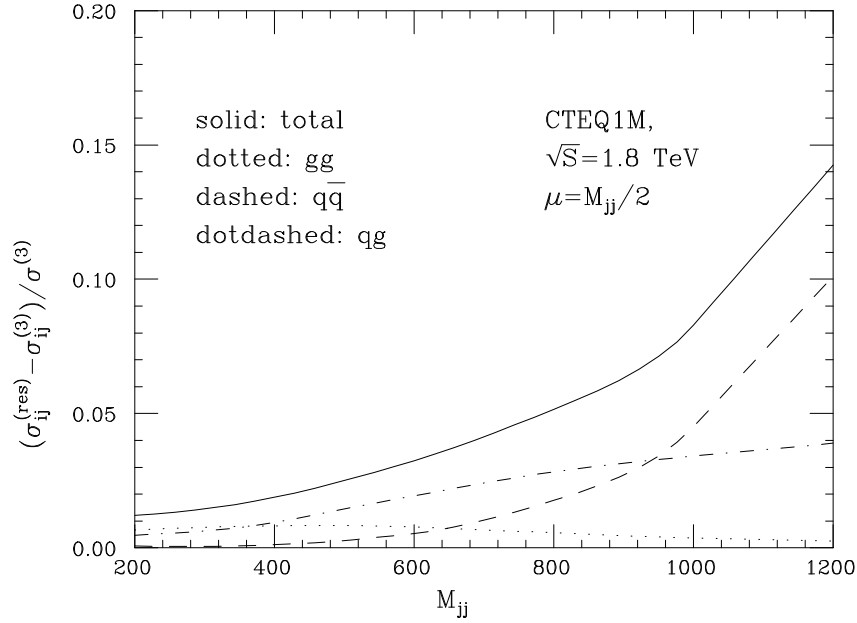


Figure 7: Contribution of gluon resummation at order  $\alpha_s^4$  and higher, relative to the truncated  $\mathcal{O}(\alpha_s^3)$  result, for the invariant mass distribution of jet pairs at the Tevatron.

## 4 Conclusions

The main conclusions of this review can be summarized as follows:

- The NLO approximation is solid up to  $E_T = 500$  GeV, with uncertainties at the level of 10–20%. The effects of higher-order corrections, estimated here in the case of dijet production by resumming the largest leading soft logarithms appearing as  $x \rightarrow 1$ , will become non-negligible only for transverse energies larger than 600 GeV.
- Modifications, at large  $x$ , of the gluon density evaluated so far seem possible without affecting the overall quality of global PDF fits. It seems possible to absorb a large fraction of the excess of high- $E_T$  jets reported by CDF, by allowing for a harder gluon at large  $x$ , without affecting the comparison of theory with other sets of data.
- While resummation corrections do not seem important in the evaluation of the hard cross sections, their effect could be non-negligible on the evolution of the valence quark densities from the low- $Q^2$  region (where they are measured) to the high- $Q^2$  region relevant for the Tevatron jets.
- The new measurements of  $F_2$  and  $xF_3$  by CCFR call for new analyses of the global fits to the PDFs. Only at that point will a new assessment of the situation with the high- $E_T$  Tevatron jets be possible.

The last two points indicate that, in spite of the significant progress we witnessed recently, there is still room for new ideas and for improvement.

## References

- [1] F. Abe et al., CDF Collaboration, *Phys. Rev. Lett.* **68**(1992)1104; *ibid.* **77**(1996)438, hep-ex/9601008.
- [2] I.A. Bertram, for the D0 Collaboration, presented at this Conference. FERMILAB-CONF-96/174-E.
- [3] R.K. Ellis and J.C. Sexton, *Nucl. Phys.* **B269**(1986)445.
- [4] F. Aversa, P. Chiappetta, M. Greco and J.P. Guillet, *Nucl. Phys.* **B327**(1989)105, *Phys. Rev. Lett.* **65**(1990)401;  
S. Ellis, Z. Kunszt and D. Soper, *Phys. Rev.* **D40**(1989)2188, *Phys. Rev. Lett.* **64**(1990)2121.
- [5] W.T. Giele, E.W.N. Glover and D.A. Kosower, *Phys. Rev. Lett.* **73**(1994)2019.
- [6] W.T. Giele and E.W.N. Glover, FERMILAB-Conf-95/169-T, in the Proceedings of the Xth Topical Workshop on Proton-Antiproton Collider Physics, R. Raja and J. Yoh eds., 1995, p. 507.
- [7] For a review and for detailed references, see J.F. Owens and W.K. Tung, *Annu. Rev. Nucl. Part. Sci.* **42** (1992)291.
- [8] H.L. Lai et al., MSU-HEP-60426, hep-ph/9606399.
- [9] A.D. Martin, R.G. Roberts and W.J. Stirling, *Phys. Lett.* **B354**(1995)155.
- [10] A.D. Martin, R.G. Roberts and W.J. Stirling, *Phys. Lett.* **B356**(1995)89.

- [11] E.N. Glover, A.D. Martin, R.G. Roberts and W.J. Stirling, *Phys. Lett.* **B381**(1996)353, [hep-ph/9603327](#).
- [12] W.T. Giele, E.W.N. Glover and J. Yu, *Phys. Rev.* **D53**(1996)120, [hep-ph/9506442](#).
- [13] S. Catani, M. Mangano, P. Nason and L. Trentadue, CERN-TH/96-86, [hep-ph/9604351](#), to appear in Nucl. Phys. B.
- [14] J. Huston et al., *Phys. Rev. Lett.* **77**(1996)444, [hep-ph/9511386](#).
- [15] D. Harris, for the CCFR Collaboration, presented at the International Conference on High Energy Physics, Warsaw, pa03-031.
- [16] For a recent review, see: A.L. Kataev, Preprint INR-0932/96, [hep-ph/9609398](#).
- [17] For a recent study, and references to previous work, see: M. Girone and M. Neubert, *Phys. Rev. Lett.* **76**(1996)3061.
- [18] Particle Data Group, R.M. Barnett et al., *Phys. Rev.* **D54**(1996)1.
- [19] For an updated review of the status of  $\alpha_s$  measurements, see: S. Bethke, PITHA 96/30, [hep-ex/9609014](#), to appear in the Proceedings of the QCD Euroconference 96, Montpellier.
- [20] M. Zielinski, for the E706 Collaboration, presented at the International Conference on High Energy Physics, Warsaw, pa03-018.
- [21] G. Curci and M. Greco, *Phys. Lett.* **102B**(1981)280.
- [22] D. Appel, G. Sterman and P. Mackenzie, *Nucl. Phys.* **B309**(1988)259.
- [23] S. Ellis, Z. Kunszt and D. Soper, *Phys. Rev. Lett.* **69**(1992)1496.
- [24] G. Sterman, *Nucl. Phys.* **B281**(1987)310.
- [25] S. Catani and L. Trentadue, *Nucl. Phys.* **B327**(1989)323, and **B353**(1991)183.
- [26] M. Klasen and G. Kramer, DESY 96-077, [hep-ph/9605210](#).